A Multi–country econometric estimation of the constant elasticity of substitution

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Objective

- Establish econometrically benchmark values for the constant elasticities of substitution that characterise Computable General Equilibrium models

- WIOD database used to econometrically estimate key parameters of the GEM-E3 model

A glance at the literature

- Several empirical studies attempted to estimate the elasticity of substitution
- Influential works include those of Arrow et al (1961) and Berndt (1976)
- Recent approaches employ time series studies (Balisteri et al, 2003; Klump et al, 2004; Antras, 2004)

A glance at the literature

- A review of the literature on the estimation of substitution elasticities reveals a confusing array of results
- Variation in results is the outcome of differentials in periods of study/ underlying hypotheses/ methods used/ data employed
- Generally observed that the elasticity estimates obtained from time-series data are significantly lower than those obtained from cross-sectional data (non-stationary, trending behavior)

WIOD and substitution elasticities

- The present paper takes a fresh look at the estimation of substitution elasticities in CES

- Distinguish between short-run and long-run elasticities using appropriate econometric techniques

- Employ pooled time series from WIOD database for the estimation of labour-capital substitution elasticities

Data

Based on the WIOD database consider six different sectors of activity

Activity	Sector	WIOD code
A1	Agriculture	AtB
A2	Mining and Quarrying & Tot. Manufacturing	C, 15t16, 17t18, 19, 20, 21t22, 24, 25, 26, 27t28, 29, 30t33, 34t35, 36t37
A3	Energy	E, 23
A4	Construction	F
A5	Market Services	50, 51, 52, H, 60, 61, 62, 63, 64, J, 70, 71t74
A6	Non market services	L,M,N,O,P

Time period:1995 – 2009. Focus on three pooled data sets for each activity

Country	Region
USA and Canada	Region 1
EU15	Region 2
China, India and Japan	Region 3

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Methods

The CES production function estimated is:

$$QV_t = \gamma\left(\delta \cdot \left(e^{\lambda_1 t} QL_t\right)^{\rho} + \left(1-\delta\right) \cdot \left(e^{\lambda_2 t} QC_t\right)^{\rho}\right)^{1/\rho}$$

where:

$$QV = \frac{VA}{VA\_P} \cdot 100,\ QL = \frac{LAB}{PL} \cdot 100,\ QC = \frac{CAP}{PC} \cdot 100,$$

$$PL = \frac{\left(LAB / H\_EMP\right)}{\left(LAB_{1995} / H\_EMP_{1995}\right)} \cdot 100,\ PC = \frac{\left(CAP / K\_GFCF\right)}{\left(CAP_{1995} / K\_GFCF_{1995}\right)} \cdot 100,$$

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Methods, Direct Approach

- Nonlinear techniques for the estimation of substitution elasticity

- Non linear approach provides less information than those proposed in the literature but:
  - exposed to less measurement errors (only the series in volumes required)
  - no needed to construct relevant unit costs (i.e. for capital or labour)
  - no misspecification error when different demand behaviour exists between individual producers

- R-package “micEconCES” (Henningsen and Henningsen, 2011)

Methods, General Approach

- Estimate the CES parameters through demand functions derived by the producer profit maximization problem:

$$\max \Pi = (PV_t)\cdot QV_t - (PL_t)\cdot QL_t - (PC_t)\cdot QC_t$$

$$s.t. \qquad QV_t = \gamma\left(\delta\cdot\left(e^{\lambda_1 t}QL_t\right)^{\rho} + (1-\delta)\cdot\left(e^{\lambda_2 t}QC_t\right)^{\rho}\right)^{1/\rho}$$

- Formulation includes:
    - the factor augmented (non – neutral) technological change
    - the Hicks (neutral) technological change
    - the exogenous rate of growth

Methods

- As a result of the maximization problem the optimal factors demand (static version) equations are derived:

$$\frac{QL_t}{QV_t} = \delta^{\sigma} \cdot \gamma^{\sigma-1} e^{(\sigma-1)\lambda_1 t} \cdot \left(\frac{PL_t}{PV_t}\right)^{-\sigma} \qquad \frac{QC_t}{QV_t} = (1-\delta)^{\sigma} \cdot \gamma^{\sigma-1} e^{(\sigma-1)\lambda_2 t} \cdot \left(\frac{PC_t}{PV_t}\right)^{-\sigma}$$

- Equations can be estimated either independently or as a system with a common parameter *β* in a log form:

$$\ln\left(\frac{QL_t}{QV_t}\right) = a_1 + \varphi_1 \, t + \beta_1 \ln\left(\frac{PL_t}{PV_t}\right) \qquad \ln\left(\frac{QC_t}{QV_t}\right) = a_2 + \varphi_2 \, t + \beta_2 \ln\left(\frac{PC_t}{PV_t}\right)$$

where $\beta_i = -\sigma_i$

- For comparison reasons further estimate:

$$\ln\left(\frac{QL_t}{QG_t}\right) = a_1 + \varphi_1 \, t + \beta_1 \ln\left(\frac{PL_t}{PG_t}\right) \qquad \ln\left(\frac{QC_t}{QG_t}\right) = a_2 + \varphi_2 \, t + \beta_2 \ln\left(\frac{PC_t}{PG_t}\right)$$

Methods

- First step to examine the properties of the time series in terms of nonstationarity and autocorrelation

- Combined Fisher/Augmented Dickey–Fuller (ADF) panel unit root tests in order to determine the order of integration of each activity:
    - ratio of labor/capital to value-added inputs
    - ratio of labor/capital to gross-output inputs
    - corresponding relative payments

- Lag selection based on the minimum Schwarz criterion

- Deterministic part also taken into account (estimation: *i*-without constant or trend *ii*- with a constant or *iii*- with a constant and trend)

Methods

- Nonstationary series integrated of *I(1)*, tested for a long-run stationary relationship with Fisher/Johansen individual test

- Depending on the results appropriate specification for each time series is employed

$$\Delta \ln\left(\frac{QL_t}{QV_t}\right) = \varphi_1 + \beta_1 \,\Delta \ln\left(\frac{PL_t}{PV_t}\right) \qquad \Delta \ln\left(\frac{QC_t}{QV_t}\right) = \varphi_2 + \beta_2 \,\Delta \ln\left(\frac{PC_t}{PV_t}\right)$$

- This specification gives the short-run elasticity of substitution

Methods

- When the series are stationary partial adjustment model in order to handle for autocorrelation is used:

$$\ln\left(\frac{QL_t}{QV_t}\right) = a_1 + \varphi_1 t + \beta_1 \ln\left(\frac{PL_t}{PV_t}\right) + \sum_{i=2}^{k} \beta_i \ln\left(\frac{QL_{t+1-i}}{QV_{t+1-i}}\right)$$

$$\ln\left(\frac{QC_t}{QV_t}\right) = a_1 + \varphi_1 t + \beta_1 \ln\left(\frac{PC_t}{PV_t}\right) + \sum_{i=2}^{k} \beta_i \ln\left(\frac{QC_{t+1-i}}{QV_{t+1-i}}\right)$$

- Short-run elasticity is $-\beta_i$ and long-run elasticity is calculated as:

$$-\beta_1 \Big/ \left(1 - \sum_{i=2}^{k} \beta_i\right)$$

Methods

- When series are both integrated of *I(1)* and cointegrated Error Correction Model (ECM) is employed:

$$\Delta \ln\left(\frac{QL_t}{QV_t}\right) = a_1 + \beta_1 \, \Delta \ln\left(\frac{PL_t}{PV_t}\right) + \beta_2 \ln\left(\frac{QL_{t-1}}{QV_{t-1}}\right) + \beta_3 \ln\left(\frac{PL_{t-1}}{PV_{t-1}}\right)$$

$$\Delta \ln\left(\frac{QC_t}{QV_t}\right) = a_1 + \beta_1 \, \Delta \ln\left(\frac{PC_t}{PV_t}\right) + \beta_2 \ln\left(\frac{QC_{t-1}}{QV_{t-1}}\right) + \beta_3 \ln\left(\frac{PC_{t-1}}{PV_{t-1}}\right)$$

- Short-run elasticity is $-\beta_i$ and long-run elasticity is calculated as:

$$\beta_3 / \beta_2$$

Estimation results
Region 1, USA CANADA

Activity	Dep. Variable	I.O.	Indep. Variable	I.O.	Method I			Method II			No. of C.E. (trace test)		LRE
					$\hat{\sigma}$	s.e.	R^2	$\hat{\sigma}$	s.e.	R^2	None	At most 1	
A1	Ln(QL/QV)	1	Ln(PL/PV)	1	0.2487	0.1144	0.1536	0.3398	0.1193	0.5086	0.0064	0.1600	1.6136
A1	Ln(QL/QG)	1	Ln(PL/PG)	1	0.2855	0.0988	0.2428	0.2911	0.0967	0.9293	0.0008	0.0704	0.7492
A1	Ln(QC/QV)	1	Ln(PC/PV)	0	0.3947	0.0847	0.3969	-	-	-	-	-	-
A1	Ln(QC/QG)	1	Ln(PC/PG)	0	0.1017	0.0638	0.0513	-	-	-	-	-	-
A2	Ln(QL/QV)	1	Ln(PL/PV)	1	0.7853	0.0845	0.7684	0.8284	0.0974	0.8650	0.0514	0.1070	1.3619
A2	Ln(QL/QG)	1	Ln(PL/PG)	1	0.5965	0.0951	0.6018	0.6074	0.0977	0.6188	0.0053	0.5734	2.8750
A2	Ln(QC/QV)	0	Ln(PC/PV)	1	0.6585	0.0689	0.7734	-	-	-	-	-	-
A2	Ln(QC/QG)	1	Ln(PC/PG)	0	0.3319	0.0615	0.5478	-	-	-	-	-	-
A3	Ln(QL/QV)	1	Ln(PL/PV)	0	0.4421	0.1645	0.1755	-	-	-	-	-	-
A3	Ln(QL/QG)	1	Ln(PL/PG)	1	0.1265	0.0952	0.0338	0.1600	0.0913	0.3168	0.0008	0.2809	0.4489
A3	Ln(QC/QV)	1	Ln(PC/PV)	1	0.6883	0.0493	0.8780	0.6794	0.0494	0.9096	0.0050	0.3976	0.5279
A3	Ln(QC/QG)	1	Ln(PC/PG)	0	0.1483	0.1007	0.0733	-	-	-	-	-	-
A4	Ln(QL/QV)	1	Ln(PL/PV)	1	0.8502	0.0874	0.8260	0.8392	0.0689	0.8844	0.0106	0.1599	1.7441
A4	Ln(QL/QG)	1	Ln(PL/PG)	0	0.7329	0.1202	0.6380	-	-	-	-	-	-
A4	Ln(QC/QV)	1	Ln(PC/PV)	1	0.3056	0.0586	0.5112	0.2107	0.0790	0.6214	0.0001	0.2612	1.3793
A4	Ln(QC/QG)	0	Ln(PC/PG)	1	0.5288	0.1738	0.4220	-	-	-	-	-	-
A5	Ln(QL/QV)	1	Ln(PL/PV)	2	0.5306	0.1128	0.4612	-	-	-	-	-	-
A5	Ln(QL/QG)	1	Ln(PL/PG)	1	0.4237	0.1001	0.4075	0.4210	0.1029	0.5882	0.0000	0.1712	0.9938
A5	Ln(QC/QV)	0	Ln(PC/PV)	1	0.4256	0.1985	0.5067	-	-	-	-	-	-
A5	Ln(QC/QG)	2	Ln(PC/PG)	1	-	-	-	-	-	-	-	-	-
A6	Ln(QL/QV)	0	Ln(PL/PV)	0	-	-	-	0.4725	0.0989	0.8714	-	-	1.0542
A6	Ln(QL/QG)	1	Ln(PL/PG)	1	0.3038	0.0903	0.3030	0.2929	0.0891	0.6268	0.0002	0.7911	0.1748
A6	Ln(QC/QV)	0	Ln(PC/PV)	1	0.0401	0.0369	0.5822	-	-	-	-	-	-
A6	Ln(QC/QG)	0	Ln(PC/PG)	1	-	-	-	-	-	-	-	-	-

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Estimation results
Region 2, EU15

Activity	Dep. Variable	I.O.	Indep. Variable	I.O.	Method I			Method II			No. of C.E. (trace test)		LRE
					$\hat{\sigma}$	s.e.	R^2	$\hat{\sigma}$	s.e.	R^2	None	At most 1	
A1	Ln(QL/QV)	1	Ln(PL/PV)	1	0.3590	0.0437	0.1995	0.3645	0.0487	0.2155	0.0001	0.1638	1.6715
A1	Ln(QL/QG)	1	Ln(PL/PG)	0	0.4072	0.0455	0.2949	-	-	-	-	-	-
A1	Ln(QC/QV)	0	Ln(PC/PV)	1	0.0852	0.0156	0.1873	-	-	-	-	-	-
A1	Ln(QC/QG)	0	Ln(PC/PG)	1	0.0398	0.0112	0.0797	-	-	-	-	-	-
A2	Ln(QL/QV)	1	Ln(PL/PV)	0	0.6016	0.0587	0.4240	-	-	-	-	-	-
A2	Ln(QL/QG)	1	Ln(PL/PG)	1	0.3121	0.0716	0.2294	0.3091	0.0766	0.1337	0.0000	0.4480	4.4166
A2	Ln(QC/QV)	0	Ln(PC/PV)	1	0.3509	0.0222	0.6315	-	-	-	-	-	-
A2	Ln(QC/QG)	1	Ln(PC/PG)	0	0.2275	0.0292	0.3052	-	-	-	-	-	-
A3	Ln(QL/QV)	1	Ln(PL/PV)	1	0.7331	0.0432	0.5801	0.7331	0.0429	0.5971	0.0077	0.3698	1.3438
A3	Ln(QL/QG)	1	Ln(PL/PG)	1	0.5044	0.0433	0.3947	0.5073	0.0432	0.4262	0.0284	0.5887	1.4296
A3	Ln(QC/QV)	1	Ln(PC/PV)	1	0.7177	0.0243	0.8250	0.7200	0.0268	0.8352	0.0000	0.1014	1.7521
A3	Ln(QC/QG)	1	Ln(PC/PG)	0	0.4308	0.0361	0.4337	-	-	-	-	-	-
A4	Ln(QL/QV)	1	Ln(PL/PV)	0	0.3827	0.0718	0.1258	-	-	-	-	-	-
A4	Ln(QL/QG)	1	Ln(PL/PG)	0	0.3291	0.0673	0.1270	-	-	-	-	-	-
A4	Ln(QC/QV)	0	Ln(PC/PV)	0	-	-	-	0.0550	0.0155	0.9935	-	-	1.2672
A4	Ln(QC/QG)	0	Ln(PC/PG)	1	0.1066	0.0191	0.3204	-	-	-	-	-	-
A5	Ln(QL/QV)	1	Ln(PL/PV)	1	0.3795	0.0543	0.2576	0.3735	0.0563	0.2654	0.0092	0.7796	2.3934
A5	Ln(QL/QG)	1	Ln(PL/PG)	1	0.4744	0.0600	0.3322	0.4842	0.0601	0.3468	0.0000	0.4944	1.5000
A5	Ln(QC/QV)	0	Ln(PC/PV)	1	0.3862	0.0276	0.6850	-	-	-	-	-	-
A5	Ln(QC/QG)	0	Ln(PC/PG)	1	0.3503	0.0599	0.4398	-	-	-	-	-	-
A6	Ln(QL/QV)	0	Ln(PL/PV)	0	-	-	-	0.1415	0.0321	0.9660	-	-	2.0656
A6	Ln(QL/QG)	0	Ln(PL/PG)	0	-	-	-	0.1664	0.0329	0.9622	-	-	2.0800
A6	Ln(QC/QV)	0	Ln(PC/PV)	1	0.0572	0.0125	0.5586	-	-	-	-	-	-
A6	Ln(QC/QG)	0	Ln(PC/PG)	1	0.0199	0.0180	0.3572	-	-	-	-	-	-

Estimation results
Region 3, China India Japan

Activity	Dep. Variable	I.O.	Indep. Variable	I.O.	Method I			Method II			No. of C.E. (trace test)		LRE
					$\hat{\sigma}$	s.e.	R^2	$\hat{\sigma}$	s.e.	R^2	None	At most 1	
A1	Ln(QL/QV)	1	Ln(PL/PV)	0	0.4732	0.0853	0.4348	-	-	-	-	-	-
A1	Ln(QL/QG)	1	Ln(PL/PG)	1	0.3343	0.0793	0.3072	0.3127	0.0813	0.3411	0.0200	0.2099	1.9247
A1	Ln(QC/QV)	0	Ln(PC/PV)	1	0.2028	0.0670	0.4355	-	-	-	-	-	-
A1	Ln(QC/QG)	0	Ln(PC/PG)	1	0.1391	0.0535	0.6244	-	-	-	-	-	-
A2	Ln(QL/QV)	1	Ln(PL/PV)	1	0.5852	0.0886	0.5212	0.5525	0.0934	0.5442	0.0000	0.2393	0.8226
A2	Ln(QL/QG)	0	Ln(PL/PG)	1	0.4180	0.1069	0.5217	-	-	-	-	-	-
A2	Ln(QC/QV)	0	Ln(PC/PV)	1	0.7142	0.0593	0.8841	-	-	-	-	-	-
A2	Ln(QC/QG)	0	Ln(PC/PG)	1	0.6151	0.0910	0.8123	-	-	-	-	-	-
A3	Ln(QL/QV)	0	Ln(PL/PV)	1	0.7496	0.0725	0.6662	-	-	-	-	-	-
A3	Ln(QL/QG)	1	Ln(PL/PG)	1	0.5250	0.0768	0.5386	0.5252	0.0718	0.6290	0.0666	0.3824	1.3559
A3	Ln(QC/QV)	0	Ln(PC/PV)	0	-	-	-	0.7126	0.0834	0.9757	-	-	1.1680
A3	Ln(QC/QG)	0	Ln(PC/PG)	1	0.5783	0.0629	0.6581	-	-	-	-	-	-
A4	Ln(QL/QV)	1	Ln(PL/PV)	1	0.7900	0.0674	0.8447	0.7328	0.0679	0.8685	0.0021	0.4420	1.6010
A4	Ln(QL/QG)	1	Ln(PL/PG)	1	0.7075	0.0873	0.7339	0.6179	0.0843	0.8159	0.0416	0.7074	2.2845
A4	Ln(QC/QV)	0	Ln(PC/PV)	1	0.2535	0.0492	0.6961	-	-	-	-	-	-
A4	Ln(QC/QG)	0	Ln(PC/PG)	1	0.2152	0.0566	0.6864	-	-	-	-	-	-
A5	Ln(QL/QV)	1	Ln(PL/PV)	1	0.7762	0.1291	0.6679	0.7913	0.1042	0.7123	0.0001	0.2447	1.1251
A5	Ln(QL/QG)	1	Ln(PL/PG)	1	0.8638	0.1285	0.7316	0.8272	0.0991	0.7243	0.0007	0.1253	1.2354
A5	Ln(QC/QV)	2	Ln(PC/PV)	1	0.4495	0.0757	0.4716	-	-	-	-	-	-
A5	Ln(QC/QG)	2	Ln(PC/PG)	1	0.3406	0.0687	0.3841	-	-	-	-	-	-
A6	Ln(QL/QV)	1	Ln(PL/PV)	0	0.7880	0.0765	0.8432	-	-	-	-	-	-
A6	Ln(QL/QG)	1	Ln(PL/PG)	0	0.9330	0.0817	0.8790	-	-	-	-	-	-
A6	Ln(QC/QV)	0	Ln(PC/PV)	1	0.1319	0.0430	0.4922	-	-	-	-	-	-
A6	Ln(QC/QG)	0	Ln(PC/PG)	1	0.0891	0.0550	0.4549	-	-	-	-	-	-

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Estimation results

- In most cases the series were found to be I(1) and cointegrated.
- Higher short run elasticities in China, India, Japan
- Higher long run elasticities in EU15
- Estimates consistent with previous empirical evidence (e.g. Berndt,1976 and Antras, 2004)
- Estimates of the elasticity based on the marginal product of labour equations tend to be higher than the estimates based on the marginal product of capital equations

Conclusions

- Short-run elasticity lower than one and sometimes close to the Leontief specification

- Long-run elasticity greater than one in most of the cases

- Longer time-series would be helpful to improve the accuracy of estimations.

- WIOD data seem to be consistent.

Thank you for your attention

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